$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto x+y$$

$$[f]^{k_1}_{k_2} = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$\ker(f) = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \vec{o} \iff x+y = \vec{o} \iff \left\{ \begin{pmatrix} x \\ -x \end{pmatrix} \middle| x \in \mathbb{R} \right\} = \text{span}\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)$$

$\text{def}(f) = 1 \implies$ není to monomorfismus

$\text{rank}(f) = 1$ (podle dimenze výchozího prostoru)

$\text{im}(f) = \mathbb{R}$ ($\text{rank}(f)$ = dimenze výstupního prostoru)

$\implies$ je to epimorfismus

$$g: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$[g]^{k_2}_{f} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$\underbrace{\mathbb{R}^3 \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}}_{f \circ g}$$

$$[f \circ g]^{k_1}_{\check{T}} = \begin{pmatrix} 3 & -3 & 3 \end{pmatrix}$$
$$\parallel$$
$$[f]^{k_1}_{k_2} \cdot [g]^{k_2}_{\check{T}} = \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 3 \end{pmatrix}$$

baze $\check{R} = ((3))$

$$[f \circ g]_{\check{T}}^{\check{R}} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} = \underbrace{T_{k_1 \to \check{R}}}_{\overset{\shortparallel}{[id]_{k_1}^{\check{R}} = \left(\frac{1}{3}\right)}} \cdot [f \circ g]_{\check{T}}^{k_1}$$

$\dim(L) = 4 \qquad M = (\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$

LZ

$\Downarrow$

$$\vec{v}_4 = \sum_{i=1}^{3} \alpha_i \vec{v}_i \Rightarrow \vec{w} \in \operatorname{span}(M) \Rightarrow \operatorname{span}(M) = \operatorname{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$$

$\Updownarrow$

$$\vec{w} = \sum_{1}^{4} \beta_i \vec{v}_i = \sum_{1}^{3} \gamma_i \vec{v}_i$$

$\dim(\operatorname{span}(M)) \leq 3$

$\Uparrow$

M negeneruje $\Rightarrow M$ je LZ

$\Updownarrow$

M generuje $\Leftarrow M$ je LNZ $\Rightarrow \dim(\operatorname{span}(M)) = 4$