$$\left(\begin{array}{ccc|c} \multicolumn{3}{c|}{A} & \vec{b} \\ 1 & a & 1 & 1 \\ a & 1 & 1 & a^2 \\ 1 & (a-1) & 1 & 1 \end{array}\right)$$

$$\begin{matrix} 1 & a & 1 \\ a & 1 & 1 \end{matrix}$$

$$\ldots \quad \det(A) = (1 + a(a-1) + a) - (1 + (a-1) + a^2)$$

$$= 1 \qquad -a \qquad =$$

$$= \underline{\underline{1-a}}$$

$$\vec{x} = \begin{pmatrix} \frac{1}{1-a} \begin{vmatrix} 1 & a & 1 \\ a^2 & 1 & 1 \\ 1 & (a-1) & 1 \end{vmatrix} \\ \frac{1}{1-a} \begin{vmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} \\ \frac{1}{1-a} \begin{vmatrix} 1 & a & 1 \\ a & 1 & a^2 \\ 1 & (a-1) & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} \frac{1}{1-a}(1-a^2) \\ 0 \\ \frac{1}{1-a}(a^2-a) \end{pmatrix} = \Big(\quad \Big)$$

$+a^2-a$

$-a^2+a$

$$\begin{vmatrix} 1 & a & 1 \\ a^2 & 1 & 1 \\ 1 & (a-1) & 1 \end{vmatrix} \quad \begin{matrix} 1 & a & 1 \\ a^2 & 1 & 1 \end{matrix} =$$

$$\begin{matrix} 1 & a & 1 \\ a & 1 & a^2 \\ 1 & (a-1) & 1 \\ 1 & a & 1 \\ a & 1 & a^2 \end{matrix} =$$

$$(1 + a^3 - a^2 + a) - (1 + a - 1 + a^3)$$

$$1 \qquad -a^2$$

$$1-a^2$$

$$\begin{matrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ a & a^2 & 1 \end{matrix}$$

$$= (a^2 + 1 + a) - (a^2 + a + 1) = 0$$

$a \neq 1$

$$\begin{pmatrix} \frac{1-a^2}{1-a} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \frac{a^2-a}{1-a} \end{pmatrix}$$

$a=1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right)\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right)\begin{matrix} R_1 \\ R_2-R_1 \\ R_3-R_1 \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = t$

$x_2 = 0$

$x_1 = 0 + x_3 =$

---

nad $\mathbb{Z}_7$

$$A = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 3 \\ 6 & 1 \end{pmatrix}$$

Jsou regulární? - det ≠ 0?

$\det(A) = (2 \cdot 3) - (2 \cdot 3) = 0$

A - není

$\det(B) = 3 \cdot 6 - 4 = 0$

B - není

$A \cdot X^T = B$ $\quad \left(\begin{array}{cc|cc} 2 & 2 & 4 & 3 \\ 3 & 3 & 6 & 1 \end{array}\right)$

$$\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 6 & 7 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 2a+2c & 2b+2d & 4 & 3 \\ 3a+3c & 3b+3d & 6 & 7 \end{array}\right)$$

$$2a+2c=4$$
$$2b+2d=3$$
$$3a+3c=6$$
$$3b+3d=7$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & 0 & 4 \\ 0 & 2 & 0 & 2 & 3 \\ 3 & 0 & 3 & 0 & 6 \\ 0 & 3 & 0 & 3 & 7 \end{array}\right) \sim \begin{array}{c} R_1 \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 5 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 3 & 0 & 3 & 7 \end{array}\right) \end{array} \begin{array}{l} \frac{1}{2}R_1 \\ \frac{1}{2}R_2 \\ R_3-R_1 \\ R_4 \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} R_1 \\ R_2 \\ R_3-R_1 \\ R_4-3R_2 \end{array}$$

$$P = \begin{pmatrix} 2 \\ 5 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{matrix} 2 \\ 5 \\ 0 \\ 0 \end{matrix} \qquad \begin{matrix} 2 & 0 \\ 5 & 0 \end{matrix}$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 3 & 0 & 0 \\ \dots \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ \dots \end{array}\right) \begin{array}{l} \frac{1}{2}R_1 \\ \frac{1}{2}R_2 \\ R_3-R \end{array}$$

$$\begin{pmatrix} 0 & 3 & 0 & 3 & | & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 & 1 & | & 0 \end{pmatrix} \begin{matrix} R_3 - R_1 \\ R_4 - R_2 \end{matrix}$$

$$\begin{array}{cccc}
a & b & c & d
\end{array}$$
$$\left(\begin{array}{cccc|c}
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)$$

$c = 1,\ d = 0$

$a + c = 0$

$b + d = 0$

$a = -c$

$b = -d$

$$\ker(A) = \operatorname{span}\left(\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$\text{řešení: } \begin{pmatrix} 2 \\ 5 \\ 0 \\ 0 \end{pmatrix} + \operatorname{span}\left(\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$\begin{pmatrix} 2 & 5 \\ 0 & 0 \end{pmatrix}^T =$$

2

$$\underbrace{\left(\begin{array}{cccc|c}
1 & 1 & 2 & 3 & 0 \\
3 & 4 & 1 & 1 & 7 \\
2 & 0 & 2 & 4 & -2 \\
2 & 5 & 1 & 0 & 9
\end{array}\right)}_{A \,|\, \vec{b}} \sim \left(\begin{array}{cccc|c}
1 & 1 & 2 & 3 & 0 \\
0 & 1 & -5 & -8 & 7 \\
0 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & -2 & 3
\end{array}\right) \begin{matrix} R_1 \\ R_2 - 3R_1 \\ R_3 - 2R_1 \\ R_4 - 2R_1 \end{matrix}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 4 & 7 & 6 \\ 0 & 0 & 2 & 3 & -2 \end{array}\right) \begin{array}{l} R_1 \\ R_3 \\ R_2 - R_3 \\ R_4 - R_3 \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 1 & 10 \end{array}\right)$$

$$\begin{array}{ccccc} a & b & c & d & \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 7 \\ 0 & 0 & -2 & -3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} \text{rank}(A) = 3 \\ \text{rank}(A|\vec{b}) = 3 \end{array}$$

$d = 0$

$-2c - 3d = 2$

$-2c = 2$

$c = -1$

$$p = \begin{pmatrix} 0 \\ 2 \\ -1 \\ 0 \end{pmatrix}$$

$b = 2$

$a = 0$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 3 & 4 & 1 & 1 & 0 \\ 2 & 0 & 2 & 4 & 0 \\ 2 & 5 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} R_1 \\ R_2 \sim \\ R_3 \\ R_4 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \\ 2 & 0 & 2 & 4 & 0 \end{array}\right. \begin{array}{l} R_1 \\ R_2 - 3R_1 \\ R \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & 5 & -1 & -4 & 0 \end{array}\right) \quad R_3 \quad R_4 - R_3$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \\ 0 & -2 & -2 & -2 & 0 \\ 0 & 5 & -1 & -4 & 0 \end{array}\right) \begin{array}{l} \cdot 2 \\ + \\ \cdot 2 \\ + \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \\ 0 & 0 & -12 & -18 & 0 \\ 0 & 5 & -1 & -4 & \end{array}\right) \begin{array}{l} \cdot(-5) \\ + \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \\ 0 & 0 & -12 & -18 & 0 \\ 0 & 0 & 26 & 36 & 0 \end{array}\right) \begin{array}{l} \cdot 26 \\ \cdot 12 \quad + \end{array}$$

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$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \end{array}\right.$$

$$\left(\begin{array}{cccc|c} 0 & 0 & -312 & -468 & 0 \\ 0 & 0 & 312 & 432 & 0 \end{array}\right) \cdot \frac{1}{...}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 1 & -5 & -8 & 0 \\ 0 & 0 & -312 & -468 & 0 \\ 0 & 0 & 0 & -34 & 0 \end{array}\right)$$

$26 \quad -39$

$$\begin{vmatrix} 5 & 1 & 0 & 3 \\ 2 & 3 & 3 & 3 \\ 4 & 0 & 0 & 6 \\ 4 & 2 & 0 & 6 \end{vmatrix} = - \begin{vmatrix} 4 & 0 & 0 & 6 \\ 2 & 3 & 3 & 3 \\ 5 & 1 & 0 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$$

$$\begin{matrix} 5 & 1 & 3 \\ 4 & 0 & 6 \\ 4 & 2 & 6 \\ 5 & 1 & 3 \\ 4 & 0 & 6 \end{matrix}$$

$$= -4 \cdot \begin{vmatrix} 3 & 3 & 3 \\ 1 & 0 & 3 \\ 2 & 0 & 6 \end{vmatrix} \begin{matrix} 3 & 3 & 3 \\ 1 & 0 & 3 \end{matrix} - 6 \begin{vmatrix} 2 & 3 & 3 \\ 5 & 1 & 0 \\ 4 & 2 & 0 \end{vmatrix} \begin{matrix} 2 & 3 & 3 \\ 5 & 1 & 0 \end{matrix}$$

$24 + 24 - 60 - 24$

$= 20 - 78 = -58$

$24 - 60$

$-36 = -1$

$(18 + 3 + 18) - (2 + 30 + 12)$

$19 \quad + 18 \quad - 30 - 12$

$$\begin{vmatrix} 0 & 1 & 5 & 3 \\ \cancel{3} & \cancel{3} & \cancel{2} & \cancel{3} \\ 0 & 0 & 4 & 6 \\ 0 & 2 & 4 & 6 \end{vmatrix} =$$

$$(2+30+12)-(12+4+15)$$

$$44-31=13$$

$$\begin{array}{r} 13 \\ \cdot 6 \\ \hline 78 \end{array}$$

3\. $$\begin{vmatrix} 1 & 5 & 3 \\ 0 & 4 & 6 \\ 2 & 4 & 6 \end{vmatrix} \begin{matrix} 1 & 5 & 3 \\ 0 & 4 & 0 \end{matrix} = \overline{24} - \overline{24} - \overline{24} = -24$$

$$\begin{array}{r} 24 \\ \cdot 3 \\ \hline 82 \end{array}$$

$= -82$

$= -5$

je regulární

$$\left(\begin{array}{ccc|c} a & -1 & 3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & a & 1 & 1 \end{array}\right)$$

$a = -1$

$$\left(\begin{array}{ccc|c} -1 & -1 & 3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & -1 & 3 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & -1 & 1 & 1 \end{array}\right) R_2 - R_1$$

$$\sim \left(\begin{array}{ccc|c} \overbrace{-1 \quad -1 \quad 3}^{A} & & & 0 \\ 0 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 1 \end{array}\right) R_3 + R_2$$

$$\left.\begin{array}{l} \text{rank}(A) = 2 \\ \text{rank}(A|\vec{b}) = 3 \end{array}\right\} f$$

$\Rightarrow$ pro $a = -1$ to nemá řešení

pro $\mathbb{R} \setminus \{-1\}$

Cramerovo pravidlo

$$\begin{vmatrix} a & -1 & 3 \\ -1 & 1 & 1 \\ 0 & a & 1 \end{vmatrix} \begin{matrix} a & -1 & 3 \\ -1 & 1 & 1 \end{matrix}$$

$a - 3a - a^2 - 1$

$-2a - a^2 - 1$

$-a^2 - 2a - 1$

$-(a^2 + 2a + 1) = -(a+1)^2$

$$\left(\begin{array}{ccc|c} a & -1 & 3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & a & 1 & 1 \end{array}\right)$$

$$\frac{1}{-(a+1)^2} \begin{vmatrix} 0 & -1 & 3 \\ 0 & 1 & 1 \\ 1 & a & 1 \end{vmatrix} \begin{matrix} 0 & -1 & 3 \\ 0 & 1 & 1 \end{matrix} = -1 - 3 = -4 \Rightarrow \left(\frac{4}{(a+1)^2} \right.$$

$$1 \quad \begin{vmatrix} a & 0 & 3 \end{vmatrix}$$

$$\frac{1}{-(a-1)^2}\begin{vmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ a & 0 & 3 \\ -1 & 0 & 1 \end{vmatrix} = -3-a \Rightarrow \begin{pmatrix} \dfrac{a+3}{(a+1)^2} \\ \\ \dfrac{1-a}{(a+1)^2} \end{pmatrix}$$

$$\frac{1}{-(a-1)^2}\begin{vmatrix} a & -1 & 0 \\ -1 & 1 & 0 \\ 0 & a & 1 \\ a & -1 & 0 \\ -1 & 1 & 0 \end{vmatrix} = a-1$$

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix} \quad -1+4-4 = -1$$

$$\begin{matrix} 1 & 4 & 0 \\ 1 & 0 & 1 \end{matrix}$$

$$\det(A) = -1$$

adj(A) = trans. matice alg. doplňků

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

$$\mathrm{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} \\ \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 1 \\ 4 & -1 & -5 \\ 4 & -1 & -4 \end{pmatrix}^T$$

$$\mathrm{adj}(A) = \begin{pmatrix} -1 & 4 & 4 \\ 0 & -1 & -1 \\ 1 & -5 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \mathrm{adj}(A) = \begin{pmatrix} 1 & -4 & 4 \\ 0 & 1 & 1 \\ -1 & 5 & 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} x_1 & x_2 & x_3 & b_1 \\ x_4 & x_5 & x_6 & b_2 \\ x_4 & x_5 & x_6 & b_3 \end{array}\right) \qquad b_1, b_2, b_3 \neq 0$$

neplatí, 2. nebo 3. řádek se může odečíst, pokud bude stejný

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det\begin{pmatrix} a-c & b-d \\ c+a & d+b \end{pmatrix}$$

$$ad - bc = (a-c)(d+b) - (c+a)(b-d)$$

$$ad - bc = ad + ab - cd - bc - cb + cd - ab + ad$$

$$2ad - 2bc$$

$$ad - bc \neq 2(ad - bc)$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$ad - bc = 0$

$eh - gf = 0 \Rightarrow eh = gf$

$\det(A+B) = 0$

$$\det\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix} = 0$$

$$(a+e)(d+h) - (c+g)(b+f) = 0$$

$$ad + ah + de + eh = bc + cf + bg + fg$$

$eh = gf$

$ad = bc$

$ah + de = cf + bg$

$ah + de - cf - bg = 0$